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# Reliability and Fuzzy Logic Concepts as Applied to Slope Stability Analysis – A Review

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## ABSTRACT
Considerable uncertainty exist with regard to stability of slopes due to several factors and recognition of these uncertainties has made designees to introduce factor of safety. Several studies, during recent years on analytical methods using soil properties have improved understanding the several uncertainties. The reliability analysis of slopes can be used to represent uncertainty in mathematical models, which can be assumed to follow the characteristic of random uncertainty. The distribution uncertain variable, which is unknown, makes its estimation difficult. Hence, the concepts of fuzzy set theory appear to be quite reliable when limited information is available. This paper attempts to review the slope stability problem and deals with the intricacies of the concept of reliability and fuzzy logic as applied to stability analysis of slope. It has been suggested that the FOSM algorithm provides a general agreement among the different slope stability solutions.
***Keywords –*** factor of safety, fuzzy logic, reliability method, slope stability,, uncertainty

## I. INTRODUCTION

Slope stability analysis includes assessing failure of an element by checking whether the stress in the element equals the strength of an element. However, the strength of soil varies from one location to another, as well as, with time. Considerable uncertainty exist with regard to stability of slopes due to several factors such as-the method of measurement of soil properties are not perfect, stress in soil is being uncertain and most of the methods used contain simplifying assumptions that leads to errors of uncertain magnitudes. Recognition of these uncertainties has made designees to introduce factor of safety, which is defined as the ratio of measured strength to calculated stress.

Risk and uncertainty in slope stability shall be dealt using observational method [1]. It shall be noted that this method will be applicable only during construction stage, where in the design can be changed. Where critical behavior cannot be observed ,the designer must rely on calculated risk[2].Several studies, during recent years on analytical methods using soil properties have improved understanding the several uncertainties. This paper attempts to summarize the slope stability problem and consider their effect in design practice and introduces the concept of reliability and fuzzy logic as applied to stability analysis of slope.

## II. PROBABILITY CONCEPTS TO SLOPE STABILITY ANALYSIS

Several investigators[3,4,5 and 6] described effective application of probability concepts for stability analysis of slopes have used first –order second moment methods(FOSM). The established methods previously deal with uncertainty regarding natural properties and loading conditions. However they do not address inadequacies such as erosion, errors in construction and inadequate soil exploration that might miss a critical geological detail. Thus, it is still better to deal relative probabilities of failure or other measures of reliability. Mean-first –order reliability method that incorporates uncertainty into decision–making process without establishing an absolute value of t he probability of failure was also used[7]. However, the probability of failure incorporates only those factors that are included in the analysis and hence considered as a lower bound of actual probability of failure. An approach in six steps to compute uncertainty in factors of safety and the corresponding reliability index as follows was recommended [2]:

(i) Identify variables such as geometry, weight and strength of soil.
(ii) To determine the mean value of each variable so onto calculate best estimate of factor of safety

(iii) Determine the uncertainty in each variable with its likely variable.
(iv) Perform sensitivity analysis to determine change in factor of safety
(v) Obtain $V [F] = \sum_{i=1}^{k} \left(\frac{\partial g}{\partial x_i}\right)^2 V[x_j] + V[e]$ (1)

where 'g' is a geometrical simplified model such as method of stress that depends on vector x of geometry and soil properties. 'e' is a modeling error that describes how a model deviates from real.

(vi) The reliability index then can be computed using $\beta = \frac{E[F] - 1.0}{\sigma [F]}$ (2)

where $\sigma [F]$ is the square root of V [ F ]
$\beta$ is the reliability index
F is the computed factor of safety.

The equation (2) gives the reliability index that describes the safety by the number of standard deviations separately, the best estimate of F from its defined failure value of 1.0. However, it shall be noted from equation (1), that the uncertainty in the model error e is uncorrelated with uncertainties in the model parameter $X_i +$ and $X_j +$

## III. RELIABILITY ANALYSIS

It was also pointed out that there is the possibility of obtaining different reliability index values for a chosen probabilistic failure surface [8]. The minimum reliability index occurs in same critical probabilistic failure surface that does not coincide with critical deterministic failure surface. Thus, the aim of reliability analysis shall be to find minimum $\beta$ value that indicates the most critical slip surface. However the reliability index can be associated with a slope but not to any specific surface.

The reliability analysis of slopes can be used to represent uncertainty in mathematical models, which can be assumed to follow the characteristic of random uncertainty. However not all uncertainties are random, especially those based on incomplete information. Hence, such uncertainties cannot be handled by probability based reliability analysis. Further, the distributions type of uncertain variable is also unknown, making its estimation rather difficult. Hence, the concepts of fuzzy set theory were recommended and appear to be quite reliable when limited information is available [9]. When the uncertainty arises due to impression and ambiguity, the variable becomes fuzzy and hence one can incorporate the fuzzy uncertainty in reliability analysis of slopes.

## IV CONCEPTS OF FUZZY SET THEORY

In this method, the soil parameters can be represented by fuzzy numbers that allows a simple and practice oriented way to allow for expert judgment [9]. The chosen membership shape can be trapezoidal one as shown in fig (1).

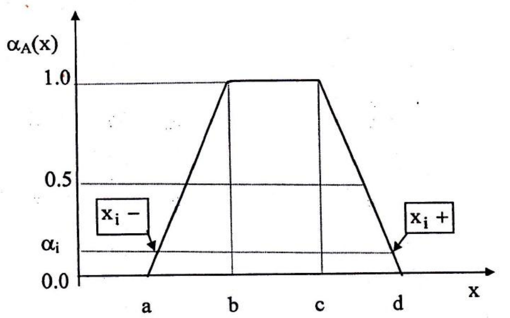


Fig. 1 Membership Shape for an uncertain parameter

The $\alpha -$ cut of a fuzzy set A can be defined as: $A_\alpha = \{ x \in X | \alpha A(x) \geq \alpha \}$, for $\alpha \in [0 ,1 ]$, i.e., the $\alpha -$ cut of a fuzzy set A is the crisp set $A_\alpha$ that contains all the elements of the universal set X hose membership grades in A are greater than or equal to the specified value of $\alpha$.

Thus the $\alpha - cut$ discretise a fuzzy member into a group of $\alpha -$ cut intervals .For each uncertain parameters , the $\alpha -$ cut gives upper and lower bound values for a particular $\alpha -$ cut.

In the fig.1, a b, c and d are calculated as:

$a = E(x_i) - m_1 \sigma(x_1)$ ;
$b = E(x_i) - m_2 \sigma(x_1)$ ;
$c = E(x_i) + m_3 \sigma(x_1)$ ;
and $d = E(x_i) + m_4 \sigma(x_1)$ ;

where $m_i$ being the number of standard deviations between expected value of the parameter and the fuzzy set vertices. Based on the expected judgments, the $m_i$ values are reported as $m_1$ = 0.5 and $m_2$ = 1.5. However, the value of $m_i$ can be modified to allow for soil behavior.

The vertex method, based on the $\alpha -$ cut concept involves interval analysis, there by reducing the fuzzy computation as a series of interval analysis [10]. The value of $\alpha$ is set to a value in the range of 0 to 1 and drawing a horizontal line that intersects fuzzy member at two points $X_i -$ and $X_i +$. Thus, the $\alpha -$ cut ranging from 0.1 to 0.995 can be considered to involve fuzzy number. For four strength parameters, there will four fuzzy numbers that can be investigated at $2 \times 4 = 8$ vertex dimension .Each of the $X_i$ vertex are treated as weak value to find the most critical failure surface[12] with the remaining parameters set at their mean values. Thus, the first order second moment (FOSM) algorithm is used to calculate minimum reliability index.

The vertex method computes two factors of safety $FS^+$ and $FS^-$. By approximate method, the derivatives for each $\alpha -$ cut is applied. The final step comprises assuming the fuzzy mean value for each derivative calculated using the following formula:

$$E\left(\frac{\partial FS}{\partial x_j}\right) = \frac{\sum_{j=1}^{N} \alpha_i W_{xi}}{N} \quad (3)$$

Where $x_i$ is the investigated possibility level

$$W_{xi} = \frac{Fs_{xi}{}^{+} + Fs_{xi}{}^{-}}{2m_i\, \sigma\,(X_i)} \quad (4)$$

where $m_i$ is the number of standard deviation of which $X_i{}^{+}$ and $X_i{}^{-}$ are greater or smaller than the mean of $X_i$ , N s the number of $\alpha -$ cut . The application of simple algorithm with correct experienced based parameter provides the result in good agreement with slope stability solutions.

## IV. CONCLUDING REMARKS

The reliability approach has evolved to a point that they can be used in actual practice but requires considerable effort and should have a good reason to use. A better and consistent approach is by a probabilistic approach based on fuzzy set theory. This approach takes into account the variability in parameters used for factor of safety calculations. Further, the FOSM algorithm provides a general agreement among the different slope stability solutions.

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